

Resummation of Large Logarithms in the Electromagnetic Form factor of π

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Abstract

In the collinear factorization of the form factor for the transition $\gamma^*\pi^0 \rightarrow \pi$ the hard part contains double log terms. These terms will spoil the perturbative expansion of the hard part. A simple exponentiation for resummation leads to divergent results. We study the resummation of these double log's. We make an analysis to show the origin of the double log's. With the understanding of the origin one can introduce soft factors and Nonstandard Light-Cone Wave Functions(NLCWF) to derive a factorized form for the form factor, where the hard part does not contain the double log's. There is a perturbative relation between NLCWF and the standard Light-Cone Wave Function(LCWF). Beside the renormalization scale μ the introduced NLCWF's and soft factors have extra scales to characterize the double log's. Using the evolutions of the extra scales and the relation we can do the resummation of the double log's perturbatively in sense that LCWF's are the only nonperturbative objects in the resumed formula. Our results with some models of LCWF show that there is a significant difference between numerical predictions with or without the resummation.

1. Introduction

Although the interaction of QCD is asymptotically weak, the perturbative theory of QCD can not directly be used to study hadronic processes involving large momentum transfers because of quark confinement. One needs to separate or factorize long-distance- and short-distance effects. Only the latter, which are characterized by large momentum transfers denoted generically as Q can be studied with perturbative QCD. For an exclusive process it has been proposed long time ago that such a process can be studied by an expansion of the amplitude in $1/Q$, corresponding to an expansion of QCD operators in twist[1, 2]. The leading term can be factorized as a convolution of a perturbative coefficient function and light-cone wave functions of hadrons. The light-cone wave functions are defined with QCD operators. The perturbative coefficient function, which can be safely calculated with perturbative QCD, describes hard scattering of partons at short distances. This is the so-called collinear factorization. In this factorization the transverse momenta of partons in parent hadrons are also expanded in the hard scattering part and they are neglected at leading twist.

Perturbative coefficient functions in collinear factorization contain large logarithms at higher orders of α_s . These large logarithms are dangerous and can spoil the expansion in the sense that the expansion does not converge. A resummation of large logarithms is often needed to have a reliable prediction. In this paper we study the resummation in the process $\gamma^*\pi^+ \rightarrow \pi^+$. The process is described by the electromagnetic form factor $F(Q)$ with Q as the virtuality of the virtual photon. For large Q the form factor can be written as a convolution

$$F(Q) \sim \phi(x) \otimes \phi(y) \otimes H(x, y) \left[1 + \mathcal{O}(\Lambda^2/Q^2) \right] \quad (1)$$

with Λ as a scale characterizing nonperturbative effects. In the above $\phi(x)$ is the light-cone wave function(LCWF) of π with the momentum fraction x carried by a parton, H is the hard part which can be calculated safely as an expansion in α_s . The one-loop correction to the hard part has been studied in [3, 4, 5, 6, 7]. It has been found that the hard part at one-loop contains double log terms like $\alpha_s^2 \ln^2 x$ and $\alpha_s^2 \ln^2 y$. At n -loop level the hard part will contain $\alpha_s^{1+n} \ln^{2n} x$ or $\alpha_s^{1+n} \ln^{2n} y$. Those double log terms will become divergent when x or y is approaching to zero and can spoil the perturbative expansion of H . Although the double log terms are integrable with x or y in the convolution and give finite contributions, but they are significant corrections. The purpose of our study is to resum those double log terms.

We study the resummation of the above double log terms in the collinear factorization by re-factorizing the double log terms in the hard part. For doing this we have to understand the origin of the double log. Given the factorized form in the above, H will receive in general contributions from the form factor and the LCWF. If we use a finite quark mass to regularize the collinear singularities, we can show that a part of double log terms come from LCWF. This part can be re-factorized by using a nonstandard light-cone wave function(NLCWF) which has a perturbative relation to LCWF. The remaining double log terms come from the form factor. They can be re-factorized into soft factors. In introducing NLCWF and soft factors non-light like gauge links play an important role. The importance of using non-light like gauge links has been shown in Transverse-Momentum-Dependent(TMD) factorization for inclusive processes[8, 9, 10, 11]. With these gauge links extra scales are introduced to control these double log terms. Using evolution equations of these scales we are able to resum the above double log terms. The outlined approach has been used to resum double log terms in $\pi\gamma^* \rightarrow \gamma$ in [12], where the resummation has a significant effect. Our approach is similar to the threshold resummation in inclusive processes studied in [13].

The resummation of these double log terms has been studied in [14] with the k_T -factorization. The k_T -factorization has been widely used in studies of B -meson decays. However, such a factorization is not gauge-invariant because hard parts are extracted from scattering amplitudes of off-shell partons. The amplitudes of off-shell partons are in general gauge-dependent. Recently it has been shown that the hard parts in the factorization receive at loop-level divergent contributions which are gauge-dependent[15]. One may find further discussions of the issue in [16]. The k_T -factorization takes transverse momenta of partons into account. These momenta are neglected at leading twist in the collinear factorization. It should be noted that the effects of the transverse momenta can be taken into account in a gauge-invariant way by using the so-called Transverse Momentum Dependent(TMD) factorization[17, 18]. It is possible to use the TMD factorization to perform the resummation. But it is at moment not possible to give numerical predictions in detail because the involved nonperturbative objects are poorly known.

Our paper is organized as the following: In Sec. 2 we introduce our notations and give a brief discussion about the consequence of the double log terms from higher orders. In Sec. 3 we explain the origin of double log terms in the hard part. In Sec. 4 we introduce NLCWF and soft-factors to re-factorize double log terms. In Sec. 5 we show that these double log terms can be resummed and give our numerical results. Sec. 6 is our conclusion. An appendix is given to discuss the problem of gauge invariance in k_T -factorization for the case studied here.

2. Notations

The electromagnetic form factor of π^+ is defined as:

$$\langle \pi^+(K) | J^\mu | \pi^+(P) \rangle = F_\pi(Q)(P + K)^\mu. \quad (2)$$

We will use the light-cone coordinate system, in which a vector a^μ is expressed as $a^\mu = (a^+, a^-, \vec{a}_\perp) = ((a^0 + a^3)/\sqrt{2}, (a^0 - a^3)/\sqrt{2}, a^1, a^2)$ and $a_\perp^2 = (a^1)^2 + (a^2)^2$. Two vectors $l^\mu = (1, 0, 0, 0)$ and $n^\mu = (0, 1, 0, 0)$ are introduced. We take a light-cone coordinate system in which the momenta are given as:

$$P^\mu \approx (P^+, 0, 0, 0), \quad K^\mu \approx (0, K^-, 0, 0), \quad q^\mu = K - P, \quad Q^2 = -q^2 \approx 2P^+K^-. \quad (3)$$

When Q^2 is very large, the form factor takes a factorized form. To derive the factorized form, i.e., to determine the hard part, one usually replaces hadronic state with partonic states to calculate the form factor and LCWF's. We replace the initial state $|\pi^+(P)\rangle$ with the partonic state $|\bar{d}(p_1)u(p_2)\rangle$, and the final state $\langle \pi^+(K) |$ with the partonic state $\langle u(k_2)\bar{d}(k_1) |$. The quark pairs are in color-singlet. The momenta are:

$$p_1^\mu = (\bar{x}_0 P^+, 0, 0, 0), \quad p_2^\mu = (x_0 P^+, 0, 0, 0), \quad k_1^\mu = (0, \bar{y}_0 K^-, 0, 0), \quad k_2^\mu = (0, y_0 K^-, 0, 0) \quad (4)$$

with $\bar{x}_0 = 1 - x_0$ and $\bar{y}_0 = 1 - y_0$. We take massless quarks and all singularities are regularized with dimensional regularization.

The contributions to the form factor of the replaced partonic states with the virtual photon attached to the u -quark are from two diagrams given in Fig.1. To reduce the number of diagrams which need to be calculated, we take $\mu = -$. Fig.1b will not contribute for $\mu = -$. It is straightforward to obtain the u -quark contribution:

$$F_\pi(Q) \Big|_{1a} = 2e_u \frac{g_s^2}{\bar{x}_0 \bar{y}_0 Q^4} \bar{v}(p_1) \gamma^\rho T^a v(k_1) \bar{u}(k_2) \gamma_\rho T^a u(p_2), \quad (5)$$

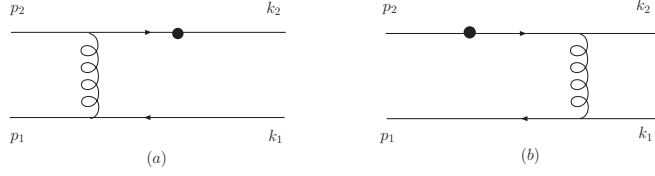


Figure 1: The leading order contributions from the u -quark. The black dot denotes the insertion of the electromagnetic current, i.e., inserting a γ^μ .

where e_u is the electric charge of the u -quark. Similarly, one can obtain the contribution from the \bar{d} -quark through the symmetry of charge conjugation.

The definition of LCWF is:

$$\phi(x, \mu) = \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle 0 | \bar{d}(0) L_n^\dagger(\infty, 0) \gamma^+ \gamma_5 L_n(\infty, z) u(z^-n) | \pi^+(P) \rangle, \quad (6)$$

where $q(x) (q = \bar{d}, u)$ is the light-quark field. L_n is the gauge link in the direction n :

$$L_n(\infty, z) = P \exp \left(-ig_s \int_0^\infty d\lambda n \cdot G(\lambda n + z) \right). \quad (7)$$

If we replace π with the parton state we have at the tree level for the wave function:

$$\phi^{(0)}(x, \mu) = \delta(x - x_0) \phi_0 + \dots, \quad \phi_0 = \bar{v}(p_1) \gamma^+ \gamma_5 u(p_2) / P^+, \quad (8)$$

where \dots stand for the states of the quark pair with quantum numbers other than that of π . For the outgoing π^+ LCWF is defined with quark fields which are separated in the direction l and we the corresponding gauge link is along the direction of l . With the definitions one easily obtains the factorized form:

$$\begin{aligned} F_\pi(Q) &\approx \frac{8\pi\alpha_s}{9Q^2} \int_0^1 dx dy \phi(x, \mu) \phi(y, \mu) \cdot \frac{1}{\bar{x}\bar{y}} \mathcal{H}(x, y, Q, \mu) \\ \mathcal{H}(x, y, Q, \mu) &= 1 + \mathcal{O}(\alpha_s). \end{aligned} \quad (9)$$

In the above the hard part \mathcal{H} can be calculated as an expansion in α_s . The correction to the factorized form factor is power-suppressed.

At one-loop level the hard part receives corrections which have double log terms[3, 4, 5, 6, 7]:

$$\mathcal{H}(x, y, Q, \mu) = 1 + \frac{\alpha_s}{3\pi} \left[\ln^2 \bar{x} + \ln^2 \bar{y} \right] + \dots, \quad (10)$$

and at higher orders of α_s^n those terms like $\alpha_s^n \ln^{2n} \bar{x}$ and $\alpha_s^n \ln^{2n} \bar{y}$, which can lead to an divergent series in α_s . If we take asymptotic form of LCWF, i.e., $\phi(x) \sim x(1-x)$, one easily finds that the $1+n$ -th order contribution to the form factor:

$$F^{(1+n)}(Q) \sim \alpha_s^{1+n} (2n)!. \quad (11)$$

The factorial increase of the $1+n$ -th order contribution can spoil the perturbative expansion. Therefore, these double log terms need to be resummed in order to have a reliable expansion. It

should be noted here that a simple exponentiation of these double log's leads to a divergent form factor because the coefficient in the front of the double log's is positive.

For the k_T -factorization in the case studied here the same diagrams in Fig.1 gives the leading-order result, in which the partons are off-shell. Therefore, the result is not gauge-invariant. We will discuss this in detail in Appendix.

3. The Origin of the Double Log Terms

In this section we analyze the origin of the double log terms in the hard part. The one-loop correction has been calculated in [3, 4, 5, 6, 7]. In [3, 7] the individual contributions from each one-loop diagram are listed in detail. From their results we find with $\mu = -$ in Eq. (2) that the diagrams given in Fig.2 give contributions containing double log's. In order to have short notations, we denote in this section the momentum fraction x_0 and y_0 in Eq.(4) as x and y , respectively.

In general the double log terms are generated if the integration region of loop momenta has an overlap between collinear- and infrared region. The infrared region always exists. The collinear region can exist in two cases. One case is that a loop momentum k is directly collinear to an external on-shell particle. In this case the integration of k will generate a collinear singularity regularized with a mass of the external particle or with dimensional regularization. Another case is that in some kinematical region. e.g. when \bar{x} or \bar{y} approaches zero, some propagators can become nearly on-shell and the collinear region is formed when k is collinear to the momenta of these propagators. The integration over k will generate the corresponding collinear singularities appearing in the form as $\ln \bar{x}$ or $\ln \bar{y}$. If there is an overlap of these collinear regions with the infrared region, double log terms will be generated. We will call those double log terms in the former case as Type I double log's, and these in the later case as Type II double log's. The existence of the two types of double log's will be explained in detail with the examples in Fig.2.

Let us first consider the contribution from Fig.2a. This contribution contains double-log terms like $\ln^2 \bar{x}$ and $\ln^2 \bar{y}$. It also contains a double pole term. These terms come from the overlap region of collinear- and soft gluons. The double-pole and double-log terms can easily be found by the approach of soft gluons where one neglects in the first step all loop momenta k in the nominator and picks the dominant terms in the denominator for $k \rightarrow 0$. Then we have the soft part from Fig.2a:

$$\begin{aligned} F_\pi(Q) \Big|_{2a} &\approx F_\pi^{(0)}(Q) \cdot \left(\frac{ig_s^2}{2N_c} \right) \int \frac{d^4 k}{(2\pi)^4} \frac{4p_1 \cdot k_1}{((p_1 + k)^2 + i\varepsilon)((k_1 + k)^2 + i\varepsilon)(k^2 + i\varepsilon)}, \\ &= F_\pi^{(0)}(Q) \left[\frac{\alpha_s}{4\pi N_c} \left(\left(\frac{2}{\epsilon} \right)^2 + \frac{2}{\epsilon} \left(-\gamma + \ln \frac{4\pi\mu^2}{\bar{x}\bar{y}Q^2} \right) + \frac{1}{2} \ln^2 \frac{4\pi\mu^2}{\bar{x}\bar{y}Q^2} \right) \right]. \end{aligned} \quad (12)$$

The reason for the double log's and the double pole in Fig.2a is the following: The exchanged gluon 1 can be collinear to the initial \bar{q} with the momentum p_1 , it can also be soft. This overlapped region produces an double pole and the double log of p_1^+ , i.e., $\ln^2 \bar{x}$. Similarly, the gluon can also be collinear to the antiquark in the final state with the momentum k_1 . Again there is an overlapped region of the soft momentum. This produces another double pole and the double log of k_1^- , i.e., $\ln^2 \bar{y}$. The sum of the two double pole contributions is given in the above. It is clearly that this type of double log's is associated with partons in the initial- or final state. According to the discussion at the beginning of this section, these double log's are Type I double log's. Actually, the double

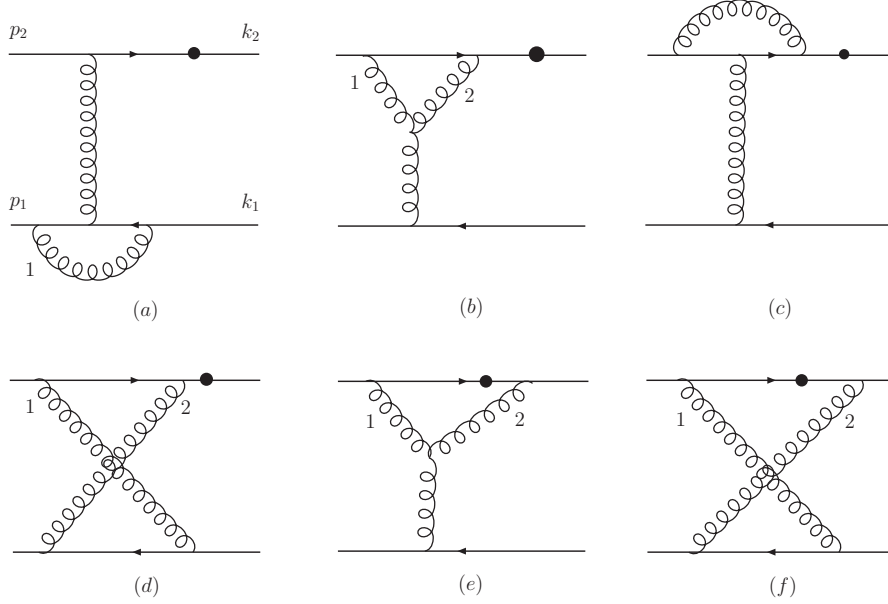


Figure 2: One-loop correction with double log terms, the original labels of the diagrams in [7] are also given.

log's and the double pole contributions from Fig.2a are canceled in the final result because the quark pair in the initial- and final state is neutral in color. We note that the soft gluon emitted by the initial \bar{q} can be absorbed by the final \bar{q} and the final q quark, the former contribution is given by Fig.2a, the later contribution is given by Fig.2f when the gluon 2 is soft. The sum of the two contributions represents the interaction of the soft gluon with the quark pair in the final state. The interaction of the soft gluon with the quark pair vanishes because the pair is color-neutral. Therefore, the double pole and double log associated with p_1 in Fig.2a are canceled by those in Fig.2f. With the same reason, one finds that the double pole and double log associated with k_1 in Fig.2a are canceled by the soft contribution from Fig.2d with the gluon 1 as the soft gluon. We will show this explicitly.

We turn to the contribution from Fig.2b. In Fig.2b any one of the two gluons from the gluon splitting can be soft. We denote the momentum of the gluon 1 as k . The gluon 2 carries the momentum $(k_1 - p_1 - k)$. If the gluon 1 is soft, we have the approximation for $\bar{x} \rightarrow 0$:

$$\begin{aligned}
 F_\pi(Q^2)|_{2b} &= \frac{g_s^4 f^{abc}}{K^-(p_1 - k_1)^2 (P - k_1)^2} \int \frac{d^4 k}{(2\pi)^4} \frac{8k_1^- k_1 \cdot p_2}{(k^2 + i\varepsilon)((k_1 - p_1 - k)^2 + i\varepsilon)} \\
 &\quad \cdot \frac{1}{-2p_2^+ k^- + i\varepsilon} \left[\bar{u}(k_2) \gamma^\alpha T^c T^b u(p_2) \bar{v}(p_1) \gamma^\alpha T^a v(k_1) \right] + \dots \\
 &= F_\pi^{(0)}(Q^2) \left\{ \frac{\alpha_s N_c}{4\pi} \left[\left(-\frac{2}{\epsilon} + \gamma + \ln \frac{\bar{x} \bar{y} Q^2}{4\pi \mu^2} \right) \ln \bar{x} - \frac{1}{2} \ln^2 \bar{x} \right] \right\} + \dots \quad (13)
 \end{aligned}$$

The term $1/(-2p_2^+ k^- + i\varepsilon)$ is the quark propagator with the soft gluon approximation. This contribution only generates the double log of \bar{x} . It also contains a collinear singularity from the region in which k is collinear to p_2 . For the case that the gluon 2 is soft, no double log term is

generated. There is also no double log term $\ln^2 \bar{y}$ for $\bar{y} \rightarrow 0$. For our purpose it is important to understand the above double log terms.

In Eq.(13) there are two terms with the double log of p_1^+ . One is associated with a collinear divergence, one is given as $-(\ln^2 \bar{x})/2$. These two terms have different origins. The initial quark emits a collinear gluon and the gluon is absorbed by the virtual gluon attached to the antiquark line. The virtual gluon becomes on-shell when \bar{x} approaches to 0. With $\bar{x} = 0$ one will find a double pole, one is collinear, another is I.R. one. In the case with $\bar{x} \neq 0$, the I.R. singularity is regularized and it generates one $\ln \bar{x}$. Another $\ln \bar{x}$ is generated from the collinear region. This is the origin for the first double log. It is clear that this double log is associated with the collinear region of p_2 and this double log is a Type I double log. The existence of Type I double log also depends on how the collinear singularity is regularized. To see this more clearly, we can use a quark mass m to regularize the collinear singularity. Then Eq.(13) becomes for $\bar{x} \rightarrow 0$:

$$\begin{aligned} F_\pi(Q^2)|_{2b} &= \frac{g_s^4 f^{abc}}{K^-(p_1 - k_1)^2 (P - k_1)^2} \int \frac{d^4 k}{(2\pi)^4} \frac{8k_1^- k_1 \cdot p_2}{(k^2 + i\varepsilon)((k_1 - p_1 - k)^2 + i\varepsilon)} \\ &\quad \cdot \frac{1}{(k - p_2)^2 - m^2 + i\varepsilon} \left[\bar{u}(k_2) \gamma^\alpha T^c T^b u(p_2) \bar{v}(p_1) \gamma^\alpha T^a v(k_1) \right] + \dots \\ &= F_\pi^{(0)}(Q^2) \left\{ \frac{\alpha_s N_c}{4\pi} \left[-\ln \frac{m^2}{\bar{y} Q^2} \ln \bar{x} - \frac{1}{2} \ln^2 \bar{x} + \dots \right] + \dots \right\}. \end{aligned} \quad (14)$$

From the above, one can see that the discussed double log disappear with the quark mass. It should be noted that the hard part does not depend how the collinear singularities are regularized. If one uses the mass to regularize the collinear singularity, one should also calculate the wave functions with the mass in order to subtract collinear singularities in the scattering amplitude. Then the Type I of double log's appearing in the hard part come from the wave functions. As we will show that Type I double log's can be factorized with NLCWF introduced in [12].

The second double log in Eq.(13), or that in Eq.(14) has a different origin than that of Type I double log's. The virtual gluon attached with the antiquark line carries the momentum $p_1 - k_1$. For $\bar{x} \rightarrow 0$, the virtual gluon approaches on-shell, its momentum is nearly collinear to $-k_1$. Here again the gluon 1 can have a momentum region which is the overlap region with the loop momentum is collinear to $-k_1$ and soft. For small non-vanishing \bar{x} the collinear singularity and the I.R. singularity are simultaneously regularized by \bar{x} , and it results in the second double log $\ln^2 \bar{x}$. This double log is a Type II double log. As we will show later, this type of double log's can be factorized by certain soft factors.

The contribution from Fig.2e contains the double log $\ln^2 k_1^-$, i.e., $\ln^2 \bar{y}$. This double log can be obtained by making the approximation for the contribution in the limit of $k \sim \delta$ and $k_1 \sim \delta$ with $\delta \rightarrow 0$, where we denote the momentum of the gluon 2 as k . Under the approximation we obtain:

$$\begin{aligned} F_\pi(Q)|_{2e} &\approx -\frac{g_s^4}{K^-} \frac{f^{abc}}{(p_1 - k_1)^2} \left[\bar{u}(k_2) \gamma^\alpha T^c T^b u(p_2) \bar{v}(p_1) \gamma^\alpha T^a v(k_1) \right] \\ &\quad \cdot \int \frac{d^4 k}{(2\pi)^4} \frac{2p_1^+}{P^+} \frac{1}{(k^2 + i\varepsilon)((k_1 - p_1 - k)^2 + i\varepsilon)(k^+ + i\varepsilon)} + \dots \\ &= F_\pi^{(0)}(Q) \left[\frac{\alpha_s N_c}{4\pi} \left(\ln \bar{y} \left(-\frac{2}{\epsilon} + \gamma - \ln 4\pi \right) + \ln \bar{y} \ln \frac{\bar{x} Q^2}{\mu^2} + \ln^2 \bar{y} - \frac{1}{2} \ln^2 \bar{y} \right) \right] + \dots \end{aligned} \quad (15)$$

Again, the first double log of \bar{y} is Type I double log, and the term $-(\ln^2 \bar{y})/2$ is Type II. Analyzing

the contribution from the region of $p_1 \sim k \sim \delta$ and the cases with the soft gluon as the gluon 1, we do not find any other double log's.

A complicated case is the contribution from Fig.2f. We first consider the case where the gluon 2 with the momentum k is soft. The contribution in the region of $p_1 \sim k \sim \delta$ we have the approximation:

$$\begin{aligned}
F_\pi(Q)|_{2f} &\approx -i \frac{g_s^4}{2N_c K^-} \left[\bar{u}(k_2) \gamma^\beta T^a u(p_2) \bar{v}(p_1) \gamma_\beta T^a v(k_1) \right] \\
&\quad \cdot \frac{2k_2^-}{P \cdot k_1} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 + i\varepsilon)((k - k_2)^2 + i\varepsilon)((k - p_1)^2 + i\varepsilon)} + \dots \\
&= F_\pi^{(0)}(Q) \left[-\frac{\alpha_s}{4\pi N_c} \left(\left(\frac{2}{\epsilon} \right)^2 + \frac{2}{\epsilon} \left(-\gamma + \ln \frac{4\pi\mu^2}{\bar{x}yQ^2} \right) + \frac{1}{2} \ln^2 \frac{4\pi\mu^2}{\bar{x}yQ^2} \right) \right] + \dots \quad (16)
\end{aligned}$$

This contribution generates the double log of $\ln^2(p_1^+)$, it also generates a double log of $\ln^2(k_2^-)$. The later is irrelevant here for our purpose. From the above one can see that the double pole and the associated double log $\ln^2 \bar{x}$, i.e., $\ln^2(p_1^+)$, is canceled by those from Fig.2a, as discussed before. The contribution from Fig.2f also contains a double log of k_1^- . To analyzing the contribution from the region of $k \sim k_1 \sim \delta$ one can use the trick in [3] to decompose the production of five propagators into a sum of products of four propagators. Then one can find the dominant contribution from the region $k \sim k_1 \sim \delta$ as:

$$\begin{aligned}
F_\pi(Q)|_{2f} &\approx i \frac{g_s^4 k_2^-}{N_c K^- p_1 \cdot k_1} \left[\bar{u}(k_2) \gamma^\beta T^a u(p_2) \bar{v}(p_1) \gamma_\beta T^a v(k_1) \right] \\
&\quad \cdot \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 + i\varepsilon)((k - k_2)^2 + i\varepsilon)((P - k_1 - k)^2 + i\varepsilon)} + \dots \\
&= F_\pi^{(0)}(Q) \left[-\frac{\alpha_s}{4\pi N_c} \left(\ln \bar{y} \left(-\frac{2}{\epsilon} + \gamma + \ln \frac{Q^2}{4\pi\mu^2} \right) + \ln^2 \bar{y} - \frac{1}{2} \ln^2 \bar{y} \right) \right] + \dots \quad (17)
\end{aligned}$$

Again, the first double log of \bar{y} is Type I double log, and the term $-(\ln^2 \bar{y})/2$. For the case that the gluon 1 is soft with $\mu = -$ one does not find the double log of $\ln^2 k_1^-$. Hence, with $\mu = -$ only the soft gluon 2 generates the wanted double log's. The above results of Fig.2f are in agreement with those in [3, 7].

Performing a similar analysis of contribution from Fig.2d, we find all relevant double log's from the region where the gluon 1 is soft. We denote k as the momentum of the gluon 1. In the region $k_1 \sim k \sim \delta$ we have the approximation:

$$\begin{aligned}
F_\pi(Q)|_{2d} &\approx -i g_s^4 \frac{2x}{N_c x Q^2} \left[\bar{u}(k_2) \gamma^\alpha T^a u(p_2) \bar{v}(p_1) \gamma_\alpha T^a v(k_1) \right] \\
&\quad \cdot \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 + i\varepsilon)((k + k_1)^2 + i\varepsilon)((p_2 + k)^2 + i\varepsilon)} \\
&\approx F_\pi^{(0)}(Q) \left[-\frac{\alpha_s}{4\pi N_c} \left(\left(\frac{2}{\epsilon} \right)^2 + \frac{2}{\epsilon} \left(-\gamma + \ln \frac{4\pi\mu^2}{x\bar{y}Q^2} \right) + \frac{1}{2} \ln^2 \frac{4\pi\mu^2}{x\bar{y}Q^2} \right) \right] + \dots \quad (18)
\end{aligned}$$

Comparing the result in Eq.(12) for Fig.2a one realizes that the double log $\ln^2 \bar{y}$ in the above is canceled by that in Eq.(12). The contributions from Fig.2c does not have relevant double log's.

Adding the soft contributions from Fig.2a, Fig.2c, Fig.2d and Fig.2f., all double poles are canceled. For the region $p_1 \sim k \sim \delta$ we similarly have:

$$\begin{aligned}
F_\pi(Q)|_{2d} &\approx ig_s^4 \frac{4p_2^+ k_1^-}{N_c Q^2 \bar{x} \bar{y} Q^2} [\bar{u}(k_2) \gamma^\alpha T^a u(p_2) \bar{v}(p_1) \gamma_\alpha T^a v(k_1)] \\
&\cdot \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 + i\varepsilon)((p_2 + k)^2 + i\varepsilon)((p_1 - k_1 - k)^2 + i\varepsilon)} + \dots \\
&\approx F_\pi^{(0)}(Q) \left[-\frac{\alpha_s}{4\pi N_c} \left(\ln \bar{x} \left(-\frac{2}{\epsilon} + \gamma + \ln \frac{\bar{y} Q^2}{4\pi \mu^2} \right) + \ln^2 \bar{x} - \frac{1}{2} \ln^2 \bar{x} \right) \right] + \dots \quad (19)
\end{aligned}$$

Again, the first double log of \bar{x} is Type I double log, and the term $-(\ln^2 \bar{x})/2$.

From the above analysis, one can find that the final contributions of double log's to the perturbative coefficient \tilde{H} only come from Type I and Type II double log's in Fig.2b, Fig.2e, Fig.2f and Fig.2d. Adding every double log's together we have:

$$\begin{aligned}
\mathcal{H}(x, y, Q, \mu) &= 1 + \frac{\alpha_s(N_c^2 - 1)}{4\pi N_c} \left[(\ln^2 \bar{x} + \ln^2 \bar{y}) - \frac{1}{2} (\ln^2 \bar{x} + \ln^2 \bar{y}) \right] \\
&+ \frac{2\alpha_s}{3\pi} \ln \bar{x} \ln \bar{y} + \frac{\alpha_s}{4\pi} \left[\beta_0 \ln \frac{\mu^2}{Q^2} - \frac{8}{3} (3 + \ln \bar{x} + \ln \bar{y}) \ln \frac{\mu^2}{Q^2} \right] + \dots, \quad (20)
\end{aligned}$$

where \dots stand for non-log, i.e., rational terms. In the first line the double log's given in $[\dots]$ come from Fig.2b, Fig.2e, Fig.2f and Fig.2d. The Type I double log's are given in the first (\dots) , the Type II double log's are given in the second (\dots) . The results in the second line are from previous results in [3, 4, 5, 6, 7]. With the above results the double log $\ln^2 \bar{x}$ from Fig.2b and Fig.2d in Eq.(19) and the double log $\ln^2 \bar{y}$ from Fig.2e and Fig.2f in Eq.(17) need to be resummed.

4. Factorization of the Double Logs

4.1 Factorization of Type I double log's

We have seen in the last section that the origin of Type I double log's depends on how the collinear divergence is regularized. As discussed in detail in [12], this type of double log's can be factorized by introducing NLCWF. The definition of NLCWF is:

$$\begin{aligned}
\tilde{\phi}(x, \zeta_{\tilde{u}}, \mu) &= \int \frac{dz^-}{2\pi} e^{ik^+ z^-} \frac{\langle 0 | \bar{q}(0) L_{\tilde{u}}^\dagger(\infty, 0) \gamma^+ \gamma_5 L_u(\infty, z^- n) q(z^- n) | \pi^0(P) \rangle}{\tilde{S}(z^-, \zeta_u)}, \\
L_{\tilde{u}}(\infty, z) &= P \exp \left(-ig_s \int_0^\infty d\lambda \tilde{u} \cdot G(\lambda \tilde{u} + z) \right), \quad \tilde{u}^\mu = (\tilde{u}^+, \tilde{u}^-, 0, 0). \quad (21)
\end{aligned}$$

In the above the limit $\tilde{u}^- \gg \tilde{u}^+$ should be taken. The main difference between LCWF and NLCWF is the gauge link in definitions. In LCWF the gauge link is obtained from $L_{\tilde{u}}$ by setting $\tilde{u}^+ = 0$. It should be noted that the limit $\tilde{u}^+ \rightarrow 0$ or $\tilde{u}^- \gg \tilde{u}^+$ is taken after integrations of loop-momenta in the definition[8, 9, 10, 11]. Otherwise the above definition is reduced to that of LCWF. The defined NLCWF depends on an extra parameter

$$\zeta_{\tilde{u}}^2 = \frac{2\tilde{u}^-(P^+)^2}{\tilde{u}^+} \approx \frac{4(\tilde{u} \cdot P)^2}{\tilde{u}^2}. \quad (22)$$

The factor $\tilde{S}(z^-, \zeta_{\tilde{u}})$ is the vacuum expectation value of a product of four gauge links. Details can be found in [12]. In [12] it has been shown that there is an interesting relation between LCWF and NLCWF. It reads:

$$\tilde{\phi}(x, \zeta, \mu) = \int_0^1 dy C(x, y, \zeta, \mu) \phi(y, \mu), \quad (23)$$

where the function C can be calculated perturbatively. The relation is in fact a factorization relation. NLCWF contains the same collinear singularities as LCWF. Therefore the function C does not contain any soft divergence. Detailed results of C at one-loop level can be found in [12]. For our purpose to do the resummation at leading order, it is enough to consider the function:

$$\frac{\hat{C}(x, \zeta, \mu)}{\bar{x}} = \int_0^1 \frac{dy}{\bar{y}} C(y, x, \zeta, \mu). \quad (24)$$

From [12] we have:

$$\begin{aligned} \hat{C}(x, \zeta, \mu) = & 1 - \frac{\alpha_s(\mu)(N_c^2 - 1)}{4\pi N_c} \left\{ \frac{1}{x} \left[-\ln^2 \bar{x} + \ln \bar{x} \ln \frac{\mu^2}{\zeta^2} + 2\text{Li}_2(\bar{x}) - \frac{\pi^2}{3} \right] \right. \\ & \left. + \frac{1}{2} \ln \frac{\mu^2}{\zeta^2 \bar{x}^2} + \frac{1}{2} \ln \frac{\mu^2}{\zeta^2 x^2} + \frac{\pi^2}{3} + 2 \right\} + \mathcal{O}(\alpha_s^2). \end{aligned} \quad (25)$$

For π in the final state moving with the momentum $K^\mu = (0, K^-, 0, 0)$ one can introduce the NLCWF $\tilde{\phi}(x, \zeta_{\tilde{v}}, \mu)$ with the replacement $\tilde{u} \rightarrow \tilde{v}$ with $\tilde{v}^\mu = (\tilde{v}^+, \tilde{v}^-, 0, 0)$ and $\tilde{v}^+ \gg \tilde{v}^-$. The parameter $\zeta_{\tilde{v}}$ is defined as:

$$\zeta_{\tilde{v}}^2 = \frac{2\tilde{v}^+(K^-)^2}{\tilde{v}^-} \approx \frac{4(\tilde{v} \cdot K)^2}{\tilde{v}^2}. \quad (26)$$

Because NLCWF and LCWF contain the same collinear singularities, one can also use NLCWF to factorize the form factor. With these NLCWF's the form factor can be factorized as:

$$F_\pi(Q) \approx \frac{8\pi\alpha_s}{9Q^2} \int_0^1 \frac{dx}{\bar{x}} \frac{dy}{\bar{y}} \tilde{\phi}(x, \zeta_{\tilde{u}}, \mu) \tilde{\phi}(y, \zeta_{\tilde{v}}, \mu) \hat{\mathcal{H}}(x, y, \zeta_{\tilde{u}}, \zeta_{\tilde{v}}, Q, \mu), \quad (27)$$

Comparing two factorizations we have the relation:

$$\hat{\mathcal{H}}(x, y, \zeta_{\tilde{u}}, \zeta_{\tilde{v}}, Q, \mu) = \hat{C}^{(-1)}(x, \zeta_{\tilde{u}}, \mu) \hat{C}^{(-1)}(y, \zeta_{\tilde{v}}, \mu) \mathcal{H}(x, y, Q, \mu), \quad (28)$$

at one-loop level. At higher orders, the relation can be a convolution with the functions C' 's. We find then

$$\begin{aligned} \hat{\mathcal{H}}(x, y, \zeta_{\tilde{u}}, \zeta_{\tilde{v}}, Q, \mu) = & 1 + \frac{\alpha_s(N_c^2 - 1)}{4\pi N_c} \left[\frac{\bar{x}}{x} \ln^2 \bar{x} + \frac{\bar{y}}{y} \ln^2 \bar{y} - \frac{1}{2} (\ln^2 \bar{x} + \ln^2 \bar{y}) \right. \\ & \left. + \frac{1}{x} \ln \bar{x} \ln \frac{\mu^2}{\zeta_{\tilde{u}}^2} + \frac{1}{y} \ln \bar{y} \ln \frac{\mu^2}{\zeta_{\tilde{v}}^2} + \ln \frac{\mu^2}{\zeta_{\tilde{u}}^2} + \ln \frac{\mu^2}{\zeta_{\tilde{v}}^2} \right] \\ & + \frac{2\alpha_s}{3\pi} \ln \bar{x} \ln \bar{y} + \frac{\alpha_s}{4\pi} \left[\beta_0 \ln \frac{\mu^2}{Q^2} - \frac{8}{3} (3 + \ln \bar{x} + \ln \bar{y}) \ln \frac{\mu^2}{Q^2} \right] + \dots, \end{aligned} \quad (29)$$

where we only give these terms explicitly: the double log terms and the scale-dependent terms. For $\bar{x} \rightarrow 0$ or $\bar{y} \rightarrow 0$ we find that in $\hat{\mathcal{H}}$ there are no Type I double log's. Therefore, the Type I double log's are factorized.

4.2 Factorization of Type II double log's

In this section we will introduce soft factors to factorize Type II double log's. We first discuss how to deal the double log from Fig.2b. As discussed before, in Eq.(13,14) the term $-1/2 \ln^2 \bar{x}$ which is divergent with $\bar{x} \rightarrow 0$ is generated from the region in which the gluon 1 is soft. The contribution from the soft gluon can be subtracted by using non-light-like gauge link[19]. The subtraction can be done by replacing the quark propagator attached by the gluon 1 with the eikonal propagator:

$$\frac{1}{(p_2 - k)^2 - m^2 + i\varepsilon} \approx \frac{1}{-2p_2^+ k^- + i\varepsilon} \rightarrow \frac{1}{-v \cdot k + i\varepsilon}, \quad (30)$$

with $v^\mu = (v^+, v^-, 0, 0)$ and $v^+ \gg v^-$. Combining factors from numerators one can identify the soft contribution as the contribution which is obtained from Fig.2b by replacing the quark line attached with the soft gluon with a gauge link along the direction v . Then one realizes that the soft contribution can be subtracted or factorized with the soft factor like

$$\langle 0 | V(x, -\infty, v) G^{a,\mu}(x) G^{b,\nu}(0) | 0 \rangle, \quad (31)$$

V is the gauge link along the direction v from $-\infty$ to x . At tree-level the soft factor is just the free propagator of gluon. This type of subtraction has been discussed in [20].

The above soft factor as it stands is not gauge invariant. To factorize the Type II double log in Fig.2b with a gauge invariant soft factor one can modify or extend the above soft factor. For this we consider the following soft factor defined with field strength tensor $G^{\mu\nu}$ and gauge links:

$$S(\tilde{k}) = \int d^4x e^{i\tilde{k} \cdot x} \text{Tr} \langle 0 | T [V(x, \infty, w) G^{-\mu}(x) V(x, -\infty, v) V^\dagger(0, -\infty, \tilde{w}) G^{-\nu}(0) V^\dagger(0, \infty, w)] | 0 \rangle (g_{\mu\nu} - n_\mu l_\nu - n_\nu l_\mu), \quad (32)$$

with $\tilde{k}^\mu = (xP^+, -K^-, 0, 0)$. The gauge link along the direction v pointing to future or starting from the past is defined as:

$$\begin{aligned} V(x, \infty, v) &= P \exp \left(-ig_s \int_0^\infty d\lambda u \cdot G(\lambda v + x) \right), \\ V(x, -\infty, v) &= P \exp \left(-ig_s \int_{-\infty}^0 d\lambda u \cdot G(\lambda v + x) \right). \end{aligned} \quad (33)$$

The three vectors for directions of gauge links are defined as the following: We take the vector v as $v^\mu = (v^+, v^-, 0, 0)$. The other two vectors w and \tilde{w} are transverse. The three vectors are chosen as:

$$v \cdot w = 0, \quad v \cdot \tilde{w} = 0, \quad w \cdot \tilde{w} = 0. \quad (34)$$

In the soft factor the limit $v^+ \gg v^-$ is taken. The reason for taking the field strength tensor $G^{-\mu}$ is because with other components of $G^{\mu\nu}$ the soft factor under the limit $v^+ \gg v^-$ can have the divergence as $1/v^-$.

At the tree-level we have:

$$S^{(0)}(\tilde{k}) = -i(N_c^2 - 1) \frac{(l \cdot \tilde{k})^2}{\tilde{k}^2}. \quad (35)$$

At one-loop level, many diagrams exist. In Fig.3 and Fig.4 diagrams for one-loop corrections are given. Some diagrams are automatically zero because of the directions of gauge links in Eq.(34).

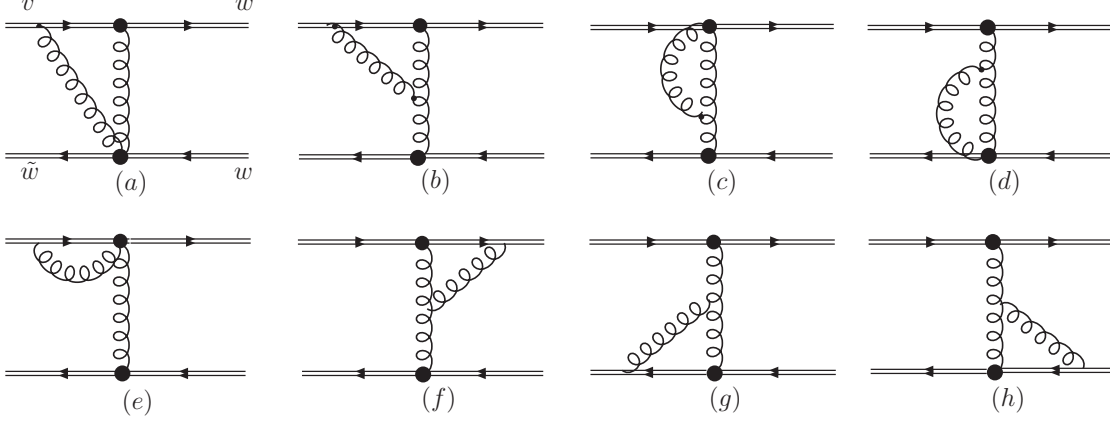


Figure 3: Part of one-loop corrections.

These diagrams are not drawn in Fig.3 and Fig.4. We first study the contribution from Fig.3b which corresponds to the soft contribution in Fig.2b. The correspondence is realized by replacing the quark propagator attached by the gluon 1 in Fig.2b with the gauge link along the direction v . It is straightforward to obtain the contribution:

$$S(q)|_{3b} = S^{(0)}(\tilde{k}) \cdot \frac{\alpha_s N_c}{4\pi} \left[-\frac{1}{2} \ln^2 \frac{\bar{x} Q^2}{\zeta_v^2} - \ln \frac{\bar{x} Q^2}{\zeta_v^2} + \frac{1}{2} \ln \left(\frac{\mu^2}{-\tilde{k}^2} \right) - \frac{2}{3} \pi^2 - \frac{1}{4} + \mathcal{O}(\zeta_v^{-2}) \right], \quad (36)$$

with

$$\zeta_v^2 = \frac{2v^+(K^-)^2}{v^-}. \quad (37)$$

From the above result one can see that the contribution from Fig.3b to the soft factor contains the same Type double log as that in the contribution from Fig.2b to the form factor, as expected.

The contribution from Fig.3e is exactly zero. The contributions from other diagrams in Fig.3 are:

$$\begin{aligned} S(\tilde{k})|_{3a} &= 0 + \mathcal{O}(\zeta_v^{-2}), \\ S(\tilde{k})|_{3e} &= 0, \\ S(\tilde{k})|_{3f} &= S(\tilde{k})|_{3g} = S(\tilde{k})|_{3h} = S^{(0)}(\tilde{k}) \frac{\alpha_s N_c}{16\pi} \left[2 \ln \left(\frac{\mu^2}{-\tilde{k}^2} \right) + 3 \right], \\ S(\tilde{k})|_{3c} &= S(\tilde{k})|_{3d} = -S^{(0)}(\tilde{k}) \frac{3\alpha_s N_c}{4\pi} \left[\ln \left(\frac{\mu^2}{-\tilde{k}^2} \right) + 1 \right]. \end{aligned} \quad (38)$$

There are also one-loop corrections from the self-energy of gauge links, exchanges of gluon between gauge links and the self-energy of gluon. The corresponding diagrams are given in Fig.4. In contrast to the contributions from Fig.3., which do not contain any soft divergence, the corrections from the self-energy of gauge links, exchanges of gluon between gauge links in Fig.4. contain I.R.

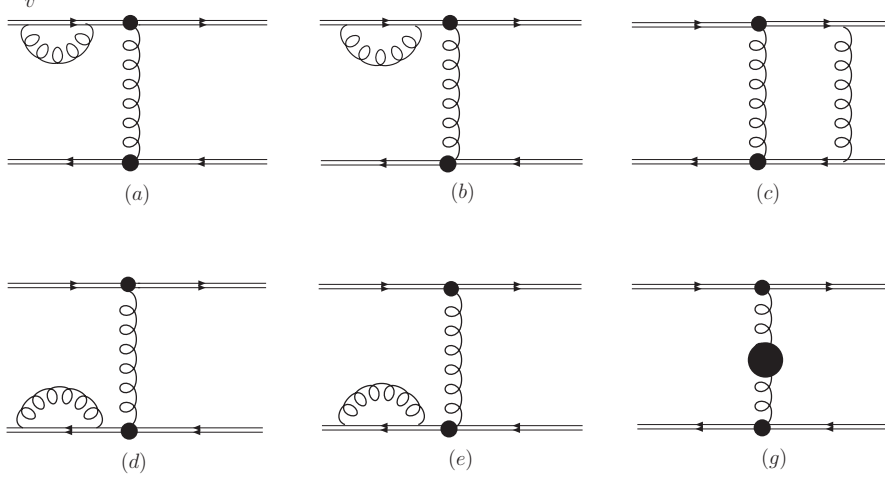


Figure 4: Self-energy-Corrections of gauge links.

singularities. These contributions can be subtracted by modifying the definition of $S(\tilde{k})$ in Eq.(32). For this purpose we introduce

$$S_0(x) = \frac{1}{N_c} \text{Tr} \langle 0 | T \left[V(x, \infty, w) V(x, -\infty, v) V^\dagger(0, -\infty, \tilde{w}) V^\dagger(0, \infty, w) \right] | 0 \rangle, \quad (39)$$

and modify $S(\tilde{k})$ to $\tilde{S}(\tilde{k})$:

$$\tilde{S}(\tilde{k}) = \int d^4x e^{i\tilde{k} \cdot x} \frac{S(x)}{S_0(x)}. \quad (40)$$

With the modification, The contributions from Fig.4a to Fig.4e are subtracted, i.e., there is no contribution from Fig.4a to Fig.4e to $\tilde{S}(\tilde{k})$. The remaining contribution from Fig.4 to $\tilde{S}(q)$ is only from Fig.4g. It is

$$\tilde{S}(\tilde{k})|_{4g} = S^{(0)}(\tilde{k}) \frac{\alpha_s}{4\pi} \left[N_c \left(\frac{5}{3} \ln \frac{\mu^2}{-q^2} + \frac{31}{9} \right) - \frac{2}{3} N_f \left(\ln \frac{\mu^2}{-q^2} + \frac{5}{3} \right) \right]. \quad (41)$$

The final result for $\tilde{S}(q)$ at one-loop level reads:

$$\begin{aligned} \tilde{S}(\tilde{k}) = S^{(0)}(\tilde{k}) & \left[1 - \frac{\alpha_s N_c}{8\pi} \left(\ln^2 \frac{\bar{x} Q^2}{\zeta_v^2} + 2 \ln \frac{\bar{x} Q^2}{\zeta_v^2} \right) - \frac{\alpha_s}{48\pi} (31 N_c + 8 N_f) \ln \frac{\mu^2}{-\tilde{k}^2} \right. \\ & \left. - \frac{\alpha_s}{4\pi} \left(N_c \left(\frac{2}{3} \pi^2 + \frac{5}{9} \right) + \frac{10}{9} N_f \right) \right]. \end{aligned} \quad (42)$$

With the soft factor \tilde{S} we can factorize the Type II double log term in Fig.2b.

Now we turn to the Type II double log $\ln^2 \bar{x}$ in Fig. 2d. It is rather difficult to construct a soft factor to factorize the double log. However, at one-loop level we can build the following soft factor which can be used for the factorization at least at one-loop level:

$$\begin{aligned} S_D(\tilde{k}) = \frac{2il \cdot \tilde{k}}{\tilde{k}^2} \int d^4x e^{i\tilde{k} \cdot x} \text{Tr} \langle 0 | T \left[V(x, \infty, n) G^{-\mu}(x) V(x, -\infty, v) V^\dagger(0, -\infty, \tilde{w}) G^{-\nu}(0) \right. \\ \left. \left(D^+ V^\dagger(0, \infty, w) \right) \right] | 0 \rangle (g_{\mu\nu} - n_\mu l_\nu - n_\nu l_\mu). \end{aligned} \quad (43)$$

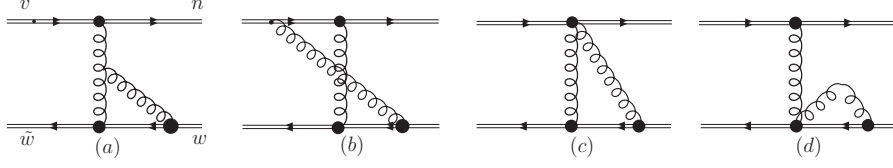


Figure 5: The leading order contributions of the correlation function S_D .

This correlation function consists of a covariant derivative of one gauge link. It is gauge invariant. The leading order contribution comes from the gluon change where the gluon is emitted by the term of the covariant derivative of the gauge link. In the limit of $v^- \rightarrow 0$ the leading order contributions to S_D come from the diagrams given in Fig.5. The contribution from Fig.5b will contain the double log corresponding to that from Fig.2d. The contribution from Fig.5d is zero in the dimensional regularization. The contributions from each diagrams in Fig.5. are:

$$\begin{aligned}
S_D(\tilde{k})|_{5a} &= S^{(0)}(\tilde{k}) \frac{\alpha_s N_c}{8\pi} \left[\ln \left(\frac{\mu^2}{-\tilde{k}^2} \right) + \frac{13}{6} \right], \\
S_D(\tilde{k})|_{5b} &= S^{(0)}(\tilde{k}) \frac{\alpha_s}{8\pi N_c} \left[\ln^2 \frac{\bar{x} Q^2}{\zeta_v^2} + 2 \ln \frac{\bar{x} Q^2}{\zeta_v^2} - \ln \left(\frac{\mu^2}{-\tilde{k}^2} \right) + \frac{4}{3} \pi^2 - 1 + \mathcal{O}(\zeta_v^{-2}) \right], \\
S_D(\tilde{k})|_{5c} &= S^{(0)}(\tilde{k}) \frac{\alpha_s N_c}{8\pi} \left[\ln \left(\frac{\mu^2}{-\tilde{k}^2} \right) + 1 \right], \\
S_D(\tilde{k})|_{5d} &= 0.
\end{aligned} \tag{44}$$

From the above result one can see that the double log $\ln^2 \bar{x}$ in the contribution from Fig.2d. to the form factor is correctly produced with the contribution from Fig.5b. to the soft factor. To factorize all Type II double log term of \bar{x} , we can now introduce the following soft factor combining above results:

$$\begin{aligned}
S_v(\bar{x} Q^2, \zeta_v, \mu) &= \frac{1}{S^0(\tilde{k})} \left(\tilde{S}(\tilde{k}) + S_D(\tilde{k}) \right) \\
&= 1 - \frac{\alpha_s (N_c^2 - 1)}{8\pi N_c} \left(\ln^2 \frac{\bar{x} Q^2}{\zeta_v^2} + 2 \ln \frac{\bar{x} Q^2}{\zeta_v^2} \right) - \frac{\alpha_s}{48\pi} \left(19N_c + 8N_f + \frac{6}{N_c} \right) \ln \frac{\mu^2}{\bar{x} Q^2} \\
&\quad + \dots + \mathcal{O}(\alpha_s^2) + \mathcal{O}(\zeta_v^{-2}),
\end{aligned} \tag{45}$$

where \dots denote constant terms. This soft factor will be used to re-factorize the double log related to q^+ , i.e., $\ln^2 x$ with q given in Eq.(10).

The Type double log of \bar{y} in the contributions from Fig.2e and Fig.2f can be handle in a similar way by introducing another soft factor. The soft factor can be obtained from S_v through the time-reversal transformation, where we replace the transformed vector v with u and the transformed momentum \tilde{k} with $(P^+, -\bar{y} K^-, 0, 0)$. The vector u is given by $u^\mu = (u^+, u^-, 0, 0)$ with $u^- \gg u^+$. We denote the obtained soft factor as $S_u(\bar{y} Q^2, \zeta_u, \mu)$. Its one-loop result can be read from S_v as:

$$S_u(\bar{y} Q^2, \zeta_u, \mu) = 1 - \frac{\alpha_s (N_c^2 - 1)}{8\pi N_c} \left(\ln^2 \frac{\bar{y} Q^2}{\zeta_u^2} + 2 \ln \frac{\bar{y} Q^2}{\zeta_u^2} \right) - \frac{\alpha_s}{48\pi} \left(19N_c + 8N_f + \frac{6}{N_c} \right) \ln \frac{\mu^2}{\bar{y} Q^2}$$

$$\begin{aligned} & + \cdots + \mathcal{O}(\alpha_s^2) + \mathcal{O}(\zeta_u^{-2}), \\ \zeta_u^2 &= \frac{2u^-(P^+)^2}{u^+}. \end{aligned} \quad (46)$$

With the introduced two soft factors we can now factorize the Type II double log's by writing the factorized form of the form factor as:

$$\begin{aligned} F_\pi(Q^2) &\approx \frac{8\pi\alpha_s}{9Q^2} \int_0^1 \frac{dx}{\bar{x}} \frac{dy}{\bar{y}} \tilde{\phi}(x, \zeta_{\bar{u}}, \mu) \tilde{\phi}(y, \zeta_{\bar{v}}, \mu) S_u(\bar{x}Q^2, \zeta_u, \mu) S_v(\bar{y}Q^2, \zeta_v, \mu) \\ &\quad \cdot \tilde{\mathcal{H}}(x, y, \zeta_{\bar{u}}, \zeta_{\bar{v}}, \zeta_u, \zeta_v, Q, \mu), \end{aligned} \quad (47)$$

with

$$\begin{aligned} \mathcal{H}(x, y, \zeta_{\bar{u}}, \zeta_{\bar{v}}, \zeta_u, \zeta_v, Q, \mu) &= 1 + \frac{2\alpha_s}{3\pi} \left[\frac{\bar{x}}{x} \ln^2 \bar{x} + \frac{\bar{y}}{y} \ln^2 \bar{y} + \ln \bar{x} \ln \frac{Q^2}{\zeta_v^2} + \frac{1}{2} \ln^2 \frac{Q^2}{\zeta_v^2} + \ln \frac{\bar{x}Q^2}{\zeta_v^2} \right. \\ &\quad + \ln \bar{y} \ln \frac{Q^2}{\zeta_u^2} + \frac{1}{2} \ln^2 \frac{Q^2}{\zeta_u^2} + \ln \frac{\bar{y}Q^2}{\zeta_u^2} + \frac{1}{x} \ln \bar{x} \ln \frac{\mu^2}{\zeta_u^2} + \frac{1}{y} \ln \bar{y} \ln \frac{\mu^2}{\zeta_v^2} \\ &\quad + \ln \frac{\mu^2}{\zeta_u^2} + \ln \frac{\mu^2}{\zeta_v^2} \Big] + \frac{83\alpha_s}{24\pi} \ln \frac{\mu^2}{Q^2} + \frac{2\alpha_s}{3\pi} \ln \bar{x} \ln \bar{y} \\ &\quad + \frac{\alpha_s}{4\pi} \left[\beta_0 \ln \frac{\mu^2}{Q^2} - \frac{8}{3} (3 + \ln \bar{x} + \ln \bar{y}) \ln \frac{\mu^2}{Q^2} \right] + \cdots, \end{aligned} \quad (48)$$

where we have taken $N_c = 3$ and $N_f = 3$. It is clear that the above expression does not contain the divergent double log $\ln^2 \bar{x}$ or $\ln^2 \bar{y}$ for $\bar{x} \rightarrow 0$ or $\bar{y} \rightarrow 0$, respectively. All these divergent double log's are contained in the soft factors and NLCWF's.

5. Resummation and Numerical Result

To re-sum those double log's in the original H we express the form factor in LCWF's through functions \hat{C} 's and take all scales at Q^2 :

$$F_\pi(Q) \approx \frac{8\pi\alpha_s}{9Q^2} \int_0^1 \frac{dx}{\bar{x}} \frac{dy}{\bar{y}} \phi(x, Q) \phi(y, Q) \hat{C}(x, Q, Q) \hat{C}(y, Q, Q) S_v(\bar{x}Q^2, Q, Q) S_u(\bar{y}Q^2, Q, Q) \mathcal{H}_r(x, y), \quad (49)$$

with the simple hard part:

$$\begin{aligned} \mathcal{H}_r(x, y) &= \mathcal{H}(x, y, Q, Q, Q, Q, Q, Q) \\ &= 1 + \frac{2\alpha_s}{3\pi} \left[\frac{\bar{x}}{x} \ln^2 \bar{x} + \frac{\bar{y}}{y} \ln^2 \bar{y} + \ln \bar{x} + \ln \bar{y} + \ln \bar{x} \ln \bar{y} \right] + \cdots, \end{aligned} \quad (50)$$

so that $\mathcal{H}_r(x, y)$ does not contain those double log's.

The resummation can be done as in the following. For \hat{C} -function we can follow [12] to express $\hat{C}(x, Q, Q)$ in term of $\hat{C}(x, \mu_0/\sqrt{\bar{x}}, \mu_0)$ with μ_0 determined by:

$$\alpha_s(\mu_0) = \bar{x}\alpha_s(Q). \quad (51)$$

It should be noted that for a given large scale Q hence a small $\alpha_s(Q)$ $\alpha_s(\mu_0)$ is always smaller than $\alpha_s(Q)$ because the asymptotic freedom of QCD. Hence an perturbative expansion of the \hat{C} function in $\alpha_s(\mu_0)$ still make sense. We have:

$$\begin{aligned}\hat{C}(x, Q, Q) &= \exp \left\{ -\frac{8}{3\beta_0} \left[\ln \bar{x} - \frac{\beta_0}{4\pi} \bar{x} \alpha_s(Q) \ln \bar{x} - \bar{x} + 1 \right] \left(\frac{\ln \bar{x}}{x} + 1 \right) \right\} \\ &\quad \cdot \hat{C}(\bar{x}, \mu_0/\sqrt{\bar{x}}, \mu_0), \\ \hat{C}(x, \mu_0/\sqrt{\bar{x}}, \mu_0) &= 1 - \frac{\alpha_s(\mu_0)}{3\pi} \left[\frac{1}{x} \left(2\text{Li}_2(\bar{x}) - \frac{\pi^2}{3} \right) - \frac{1}{2} \ln x + \frac{\pi^2}{3} + 2 \right] + \mathcal{O}(\alpha_s^2).\end{aligned}\quad (52)$$

All double log's are now re-summed in the exponential factor.

For the resummation of the soft factor S_v we have the following ζ -evolution equation:

$$\frac{\partial S_v(\bar{x}Q^2, \zeta_v, \mu)}{\partial \ln \zeta_v^2} = -\frac{2\alpha_s}{3\pi} \left(\ln \frac{\zeta_v^2}{\bar{x}Q^2} - 1 \right) S_v(\bar{x}Q^2, \zeta_v, \mu). \quad (53)$$

Using this equation we can express $S_v(\bar{x}Q^2, Q, Q)$ in term of $S_v(\bar{x}Q^2\sqrt{\bar{x}}Q, Q)$:

$$\begin{aligned}S_v(\bar{x}Q^2, Q, Q) &= \exp \left[-\frac{\alpha_s(Q)}{3\pi} (\ln^2 \bar{x} + 2 \ln \bar{x}) \right] S_v(\bar{x}Q^2\sqrt{\bar{x}}Q, Q), \\ S_v(\bar{x}Q^2, \sqrt{\bar{x}}Q, Q) &= 1 - \alpha_s(Q)(\cdots) + \mathcal{O}(\alpha_s^2),\end{aligned}\quad (54)$$

where (\cdots) contains only constant terms, i.e., no log's. Doing the same for S_u we finally have the resummed form factor:

$$\begin{aligned}F_\pi(Q) &= \frac{8\pi\alpha_s(Q)}{9} \int_0^1 \frac{dx}{\bar{x}} \frac{dy}{\bar{y}} \phi(x, Q) \phi(y, Q) e^{-S(\bar{x}, Q) - S(\bar{y}, Q)}, \\ S(\bar{x}, Q) &= \frac{8}{3\beta_0} \left[\ln \bar{x} - \frac{\beta_0}{4\pi} \bar{x} \alpha_s(Q) \ln \bar{x} - \bar{x} + 1 \right] \left(\frac{\ln \bar{x}}{x} + 1 \right) + \frac{\alpha_s(Q)}{3\pi} (\ln^2 \bar{x} + 2 \ln \bar{x}),\end{aligned}\quad (55)$$

since we work at leading order re-summation, we have to take \mathcal{H}_r at tree-level, i.e., $\mathcal{H}_r = 1$. The above formula is applicable when one knows LCWF's at the large scale Q . If LCWF's are determined in a lower scale μ , one can use evolution equation to express $\phi(x, Q)$ in $\phi(x, \mu)$. In this case we have:

$$\begin{aligned}F_\pi(Q) &= \frac{8\pi\alpha_s(Q)}{9} \int_0^1 \frac{dx}{\bar{x}} \frac{dy}{\bar{y}} \phi(x, \mu) \phi(y, \mu) e^{-S(\bar{x}, Q) - S(\bar{y}, Q) + \mathcal{K}(x, y, Q, \mu)}, \\ \mathcal{K}(x, y, Q, \mu) &= -\frac{8}{3\beta_0} \ln \frac{\alpha_s(Q)}{\alpha_s(\mu)} (3 + \ln(\bar{x}\bar{y})).\end{aligned}\quad (56)$$

We will use our resummation formula in Eq.(55) and Eq.(56) to give our numerical results. In our formulas the nonperturbative input is the LCWF. The LCWF has the asymptotic form if μ goes to ∞ :

$$\phi(x, \mu) = 6x(1-x)f_\pi + \cdots, \quad (57)$$

where \cdots stand for terms which are zero in the limit $\mu \rightarrow \infty$. The LCWF can be expanded with Gegenbauer polynomials[1]. A model for ϕ has been proposed by truncating the expansion[21]:

$$\phi(x, \mu) = 6f_\pi x(1-x) \left(1 + \phi_2(\mu) C_2^{3/2}(2x-1) \right), \quad (58)$$

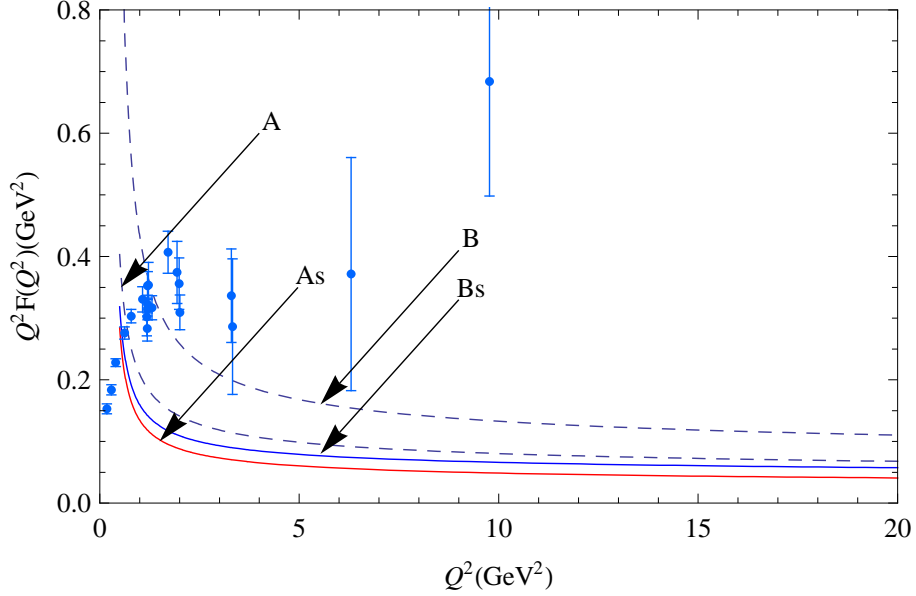


Figure 6: Numerical results for the form factor. Curve- A and Curve- A_S are drawn by using LCWF in Eq.(57) without and with the resummation. Curve- B and Curve- B_S are drawn by using LCWF in Eq.(58) without and with the resummation.

where $\phi_2(\mu_0)$ is determined by QCD sum-rule method at $\mu_0 = 1\text{GeV}$ [21]:

$$\phi_2(\mu_0 = 1\text{GeV}) = 0.44. \quad (59)$$

We will use these two types of LCWF to give our numerical results. We will use Eq.(55) with the asymptotic form of ϕ to make our numerical predictions. For LCWF given in Eq.(58) we use Eq.(56). We take the Λ -parameter as $\Lambda = 237\text{MeV}$. The numerical results are represented in Fig.6.. Our numerical results do not strongly depend on the value of Λ . There is only a little change if we change Λ from 100MeV to 300MeV .

Our numerical results show that the resummation has significant effects. For the asymptotic LCWF the difference between the resummed- and unresummed form factor can be from about 30%. The difference with LCWF in Eq.(58) is about 50%. We notice that the resummed form factor is in general smaller than the unresummed. In [7] a detailed study for numerical predictions at one-loop level has been done. With various models of LCWF's it has been shown that the one-loop correction is in general positive. The correction is large and can be at order from 40% to 100%. This brings up the question if the perturbative expansion is reliable. Because our resummed form factor becomes smaller, we expect that the situation can be improved, or at least partly improved. But this needs a detailed study by including higher order corrections.

In Fig.6 we also give the experimental result from [22]. In the case of $\gamma^*\pi \rightarrow \gamma$ we have found that the experimental data in the region $3\text{GeV}^2 < Q^2 < 10\text{GeV}^2$ can be described fairly well with the resummed form factor[12]. But in the case here, we fail to re-produce the experimental results. The similar situation also appears in the study at one-loop level in [7], where the experimental results can not be reproduced with various models of LCWF. This problem deserves a further study.

6. Conclusion

The hard part at one-loop level in the collinear factorization for the form factor in $\gamma^*\pi \rightarrow \pi$ contains double log terms of \bar{x} and \bar{y} . At n -loop level it is expected that the large log terms like $\ln^{2n} \bar{x}$ or $\ln^{2n} \bar{y}$ appear. These double log's are dangerous and can result in that the perturbative expansion of the hard part becomes a divergent expansion. A resummation of these terms with a simple exponentiation can not be done because it results in divergent results. In this work we have studied the resummation of these double log terms. For this purpose we have identified the origin of the double log's in detail in the first step. In the second step we have employed the concept of factorization. We have re-factorized the form factor so that the hard part does not contain double log's. In the re-factorization NLCWF instead of LCWF and soft factors are introduced to capture these double log terms.

Non light-like gauge links are used in NLCWF and soft factors. These links play important role in the re-factorization and resummation. Because the gauge links are not light-like, the introduced NLCWF and soft factors contain extra scales beside the renormalization scale μ . Using evolutions of these scales we are able to resum the double log terms into two exponential factors, one is for $\ln^2 \bar{x}$, while another is for $\ln^2 \bar{y}$. An interesting aspect of our approach for the resummation is that the resummed form factor only contains LCWF's as nonperturbative quantities. Every ingredient in the resummation except LCWF's can be calculated perturbatively.

With the resummation we have given numerical results for the form factor with two choices of LCWF. We have found that the resummation has significant effects. Between the resummed-the unresummed form factor the numerical difference is at level about 30% – 50%. With various models of LCWF's it has been shown in [7] that the one-loop correction is in general positive. The correction is large and can be at order from 40% to 100%. Because of the large correction, the perturbative prediction can be unreliable. With the structure of the exponentials for the resummation one can find that the resummed form factor becomes smaller than the unresummed. With this fact we expect that the situation with the perturbative expansion can be improved, or at least partly improved. But this needs a detailed study by including higher order corrections.

In the case of $\gamma^*\pi \rightarrow \gamma$ we have found that the experimental data in the region $3\text{GeV}^2 < Q^2 < 10\text{GeV}^2$ can be described fairly well with the resummed form factor[12]. It is frustrated in the case of $\gamma^*\pi \rightarrow \pi$ if one compares theoretical results of collinear factorization with experimental data. With or without resummation, the experimental data at higher Q^2 is not in agreement with perturbative results. This deserves a further study in experiment and in theory. In this work we have performed the resummation at one-loop level. It is possible to extend our work to the resummation of the remaining single log terms and beyond one-loop level.

Appendix: Gauge Variance in the k_T -Factorization

In the collinear factorization the transverse momenta of partons entering hard scattering are neglected at leading twist. As we have seen that the perturbative part in Eq.(9) contains double log's at higher order and is divergent when the momentum fraction \bar{x} or \bar{y} approaches zero. It has been suggested that for small longitudinal momenta one may need to take the transverse momenta into account. This leads to the so-called k_T -factorization. The k_T -factorization has been widely used in exclusive B -decays. Although it has been widely used, the factorization has not been studied beyond leading order except the case with $\pi\gamma^* \rightarrow \gamma$ [23] in which the study is perform with Feynman

gauge. Because the perturbative parts in the factorization are extracted from scattering amplitudes of off-shell partons and scattering amplitudes of off-shell partons are not gauge-invariant, it is not expected that the perturbative parts are gauge-invariant. In [15] it has been pointed out that such a factorization is gauge-dependent because the perturbative parts contain soft divergences at loop-level which depend on gauges. It should be noted that in the case with $\pi\gamma^* \rightarrow \gamma$ the problem of gauge invariance appears beyond tree-level, because gluons are exchanged at one- or higher loops. Taking the case with $\pi\gamma^* \rightarrow \pi$, it is easy to show the problem at tree-level, because at that level gluon-exchange happens.

In the k_T -factorization, the form factor in Eq.(2) can be factorized in a similar way as in Eq.(9):

$$F_\pi(Q) \sim \int dx d^2\tilde{k}_{1\perp} d\tilde{y} d^2k_{2\perp} \phi(x, k_{1\perp}) \phi(y, k_{2\perp}) H(x, y, k_{1\perp}, k_{2\perp}). \quad (60)$$

The definition of the k_T -dependent wave functions can be found in [15, 17, 23]. The perturbative part H is extracted from scattering of off-shell partons. Instead of parton momenta in Eq.(4) one has the momenta for these off-shell partons:

$$p_1^\mu = (\bar{x}_0 P^+, 0, \vec{p}_\perp), \quad p_2^\mu = (x_0 P^+, 0, -\vec{p}_\perp), \quad k_1^\mu = (0, \bar{y}_0 K^-, \vec{k}_\perp), \quad k_2^\mu = (0, y_0 K^-, -\vec{k}_\perp). \quad (61)$$

At leading order the same diagrams in Fig.1. give contributions to the form factor and hence to H . In calculating these one makes the projection by replacing the spinor products like those in Eq.(5) with:

$$u(p_1)\bar{v}(p_2) \rightarrow \gamma^- \gamma_5, \quad v(k_1)\bar{u}(k_2) \rightarrow \gamma_5 \gamma^+. \quad (62)$$

We take $\mu = -$ as before. Then the contribution to the form factor from the u -quark is only from Fig.1a.. If the k_T -factorization respects the gauge invariance, this contribution must be gauge-invariant. In Feynman gauge one finds the hard part from the u -quark:

$$H(x, y, p_\perp, k_\perp)|_u = \frac{1}{\bar{x}\bar{y}Q^2 + (\vec{p}_\perp - \vec{k}_\perp)^2}. \quad (63)$$

In deriving this result one has used the power counting: $\bar{x}Q \sim \bar{y}Q \sim p_\perp \sim k_\perp \sim \delta$ and only the leading term in δ has been taken into account. To extract H one has also used the tree-level result of k_T -dependent wave functions with the off-shell partons:

$$\phi(x, k_{1\perp}) \sim \delta(x - x_0) \delta^2(\vec{k}_{1\perp} - \vec{p}_\perp), \quad \phi(y, k_{2\perp}) \sim \delta(y - y_0) \delta^2(\vec{k}_{2\perp} - \vec{k}_\perp). \quad (64)$$

It should be noted that there is no problem of gauge-invariance for the tree-level result of wave functions, because there is no gluon exchange.

Now we calculate the hard part in an axial gauge fixed with an arbitrary vector w , i.e., $w \cdot G = 0$. The gluon propagator in this gauge reads:

$$\frac{-i}{q^2 + i\varepsilon} \left[g^{\mu\nu} - \frac{w^\mu q^\nu + w^\nu q^\mu}{w \cdot q} + w^2 \frac{q^\mu q^\nu}{(w \cdot q)^2} \right]. \quad (65)$$

In the above the first term is just the propagator in Feynman gauge. We obtain the contribution from the u -quark to the hard part in the axial gauge as:

$$\begin{aligned} H(x, y, p_\perp, k_\perp)|_u = & \frac{1}{\bar{x}\bar{y}Q^2 + (\vec{p}_\perp - \vec{k}_\perp)^2} \left\{ 1 - \frac{w_\perp \cdot (p - k)_\perp}{w \cdot (p_1 - k_1)} + w^2 \frac{p_\perp \cdot (p - k)_\perp}{2(w \cdot (p_1 - k_1))^2} \right. \\ & \left. + \frac{1}{2w \cdot (p_1 - k_1)} \left[\frac{l \cdot w}{\bar{y}K^-} k_\perp \cdot (k - p_\perp) + w \cdot k_\perp \right] \right\}. \end{aligned} \quad (66)$$

In this result the momentum fraction x_0 in p_1 and y_0 in k_1 in Eq. (61) should be replaced with x and y , respectively. In the above $\{\dots\}$ every term is at order of $\mathcal{O}(\delta^0)$. Therefore no term can be neglected with the power counting. This result clearly indicates that H is gauge-dependent. It is interesting to note that the first two terms are canceled and all terms in H depend on w , if we take the vector w as $w^\mu = (0, 0, w_\perp)$. For showing this clearly we give the explicit result for this case:

$$H(x, y, p_\perp, k_\perp)|_u = \frac{1}{\bar{x}\bar{y}Q^2 + (\vec{p}_\perp - \vec{k}_\perp)^2} \left\{ \frac{w_\perp^2 p_\perp \cdot (p - k)_\perp}{2(w_\perp \cdot (p_\perp - k_\perp))^2} + \frac{w_\perp \cdot k_\perp}{2w_\perp \cdot (p_\perp - k_\perp)} \right\}. \quad (67)$$

One can also show that the hard part is gauge-dependent in the general covariant gauge[24]. In the gauge the gluon propagator reads:

$$\frac{-i}{q^2 + i\varepsilon} \left[g^{\mu\nu} - \alpha \frac{q^\mu q^\nu}{q^2 + i\varepsilon} \right]. \quad (68)$$

In this gauge the contribution from the u -quark reads:

$$H(x, y, p_\perp, k_\perp)|_u = \frac{1}{\bar{x}\bar{y}Q^2 + (\vec{p}_\perp - \vec{k}_\perp)^2} \left(1 + \frac{\alpha}{2} \frac{p_\perp \cdot (p_\perp - k_\perp)}{\bar{x}\bar{y}Q^2 + (\vec{p}_\perp - \vec{k}_\perp)^2} \right). \quad (69)$$

Here the gauge-dependent term in (\dots) is proportional to α . This term is at order of $\mathcal{O}(\delta^0)$ in comparison with the first term. Therefore, the gauge-dependent term can not be neglected. One notices here that the transverse momenta appear in the numerator. One may argue that these terms with the transverse momenta in the numerator may be factorized with higher-twist operators other than the leading-twist operator used to defined $\phi(x, p_\perp)$. Even if one can do so, these terms contributing to the factorization hence to the form factor are still gauge-dependent and can not be neglected with the power counting in comparison with the term factorized with $\phi(x, p_\perp)$. The conclusion here is that the k_T -factorization here at tree-level is gauge-dependent. Beyond tree-level, the hard part will receive gauge-dependent contributions which are divergent[15, 16].

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